Odds/Ends RE Math

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1. WHY MATHEMATICS?

The mathematical point of view seems a very special one, at first. It seems but one of many sides of life. Yet Augustine called theology the supreme mathematics, and the soul a moving number. To see one's relation to another in God, is to see them as pure number, he said. Such a point of view makes mathematics the orienting heart of all things. Yet ordinary life has so little to do with mathematics. How can this be?

One key to understanding how this can be, is the recognition that "ordinary life" although not (consciously) within a mathematical system, is consciously "trying" to come within one. The biggest or hardest part of life is discovering what "the question" is. Once a question is defined, it can be tossed to system men, mathematicians and physicists, etc., to solve. Any fool can solve a problem — the real suffering is what goes into defining it.

A problem is not defined until it is recognized as part of a mathematical system. Everything which we call "reasoning, " depends upon relation to such a system. If life has taught us anything, it has taught us how dangerous and ambiguous were the various "self-evident" notions not thus tied down, how inadequate they prove as a ground for social action, at least in the modern context. Nothing is so dangerous as the "self-evident" that is not formalized (or pretends not to need to be).

Much of the feeling of frustration with the mathematical point of view is removed, when one recognizes its intimate relation to art. The struggle to reach a "mathematical" experience of our own subjectivity, to push feeling to a mathematical formulation, is, in a sense, the guiding motivation of modern art. This can be

especially seen in modern sculpture, whose break with the immediate past has been most dramatically radical.

The opening up of system goes in stages, as Wölfflin and others have pointed out. When the old system becomes inadequate, random search is reduced by just letting it fall apart, by a concentration on the momentary and spontaneous. This creates a host of undigested differences which in their turn create conflicts which press us on to a deeper unification. (Every felt difference bespeaks a "hate" from whose resolution it emerges.) Values in art then shift from momentary and personal to common and universal. But this shift is not deliberate. It is really the outcome of the former emphasis pushed on to its extreme.

When the consequences of the new layer of individuation are internalized, they lead to paralyzing separations, which set art off searching for what is common again, trying to reduce the new material (draw in in the "romantic" phase) to system.

Word systems are for richer and more flexible things than "outsiders" suspect. Rather than being a threat to individuality, and individual difference, they make possible a vast richness of individual difference, which would else be rejected as unlivable.

A society can only accept or use as much individual difference as it can make a system of. The fantastic richness of modern mathematics, and the corresponding fantastic variety of character types that "inhabit" its various problem ranges, opening them out, quickly dissipates any notion that "system-like" means "same" or

"conformist. " Rather it is standardization and the system which alone enfranchises difference.

Any difference in elaboration carries with it cascades of consequences which force either extinction, or acceptance into a common system. Or more correctly, which force in such qualifications as allow of incorporation into a common system, for no difference comes into a stable existence unless it has some root in truth somewhere.

The unification stage of art today has certain rather special characteristics, because the word has, at last, voided the eye, and taken up into itself the whole of its felt, or preverbal content. The unification stage involves a "complete" inner dissolution, followed by a reconstruction of the inner world from its elementary "symbolic" beginnings.

Cezanne was, in a sense, the father of this modern art. He saw that the struggle to experience form in its inwardness, to experience it in a way that held up under long periods of inner probing, was driving that form towards an elementary "mathematical" organization of planes, which reinforced one another in such a way as to give a sense of plenitude or solidity.

The subjective exists for us as operator, generating the motor and verbal ordering. All subjective difference is thus related to ordering difference, and, as such, is "mathematical. "

This does not allow us to limit the subjective ahead of time, any more than it allows us to limit mathematical system. As Andre Breton has said, no limits can be set to the human imagination, a technical point, with immediate counterparts within

mathematics. The fact that no such limit can be set is no bar to "mathematicalization" relative to the class of issues of present concern.

Such a mathematicalization of feelings takes many forms, depending on the special emphasis of the individual. In some, the concern is for harmony (as with Gabo) and solidity (such as the cubists looked to). With others, the concern is more dynamic (Boccioni) or vitalistic. But all such "extremes" are (as Apollinaire points out) part of a common modern tendency, the struggle to mathematicalize feeling — which is the formal side of the inner struggle to reach a common universal via which man can be linked to man once again. Art is as varied as mathematics. The mathematicalization of feeling does not mean that they are made static. It only means that they are there made adequate to a given task. When that task is done, the old resolution falls apart again. Feeling, as Boccioni says, is inherently transcendental. There is a gulf between feeling, as generator of the verbal order, and words, as the order generated, that our meta-words never fully penetrate. The felt operators never have the status of simples in our theories. It is the false expectation that they should have that, in a sense, delayed the recognition of what was happening in art.

Modern times partake of all times. Ruskin had felt that the sort of symbolic art which the Egyptians and Assyrians realized could never be relevant again, as such art depended upon the presence of an absolute authority guiding elaboration. But there are many areas of modern life more deeply and militantly "conformist" than anything Egypt dreamed of. To be a good physicist or mathematician involves a type of devotion to the authority of higher laws, a service in their elaboration, that partakes of much of this old symbolic spirit. (Past history (really) lives on as part of the present, (Croce).)

Modern life as an arena for the formation of new tasks, can make use of all the styles and energies of the past, whose differences are merely reflective of different stages or phases of task formation.

Art as so broadly conceived, in its historical panorama, becomes a symbol not for a part, but for the whole of life, a recognition which the Dada-ists represented. "Difficulty" is itself something special and orienting, like a ridge that must be passed, but all of life contributes to the new. In a dance or art class, in the "bad" art of fellow students, one touches forms of a living relevance that no dance group or museum can impart. All play essential roles sustaining and guiding the emergence of the new. Even destruction is an essential part of creation, (Bakunin).

Harmony if it is a symbol of the best, is also a symbol of the worst, (as are all the central guides). Plato's dislike of the art of his time, ideal beauty residing, he said, in cube, sphere, and cylinder, is in a sense ultimately justified by modern art, perhaps especially sculpture. Ruskin, too, found himself bored by the Greek inspired sculpture of his day. The Greek balance, the Greek ideal, has had such furious support (Wilenski), in part because of its corrupting influence, on which Plato comments. It is an art that stops at person-relatedness—whose balance is an inter-person balance, the essence of corruption in the deeper sense, (See Kierkegaard). "The half-way house of old reasons" (Boehme).

Pushing on to a constructivist view, or pure symbolic view, involves grasping the symbol felt for longer time-constants, and individuating it more precisely to the given situation, in order to have anything left. The modern preference for working in hard materials (as against clay) is a reflection of the need for longer struggles with the

form to stabilize it. The working characteristics of a chosen medium are always wanted for their own sake, never entered into "in spite," because of secondary (e. g., mere textural) values.

Michelangelo and Picasso have variously commented on the central place of sculpture in the arts. There is something overly civilized and passive about the visual which is not also sculptural, evoking the haptic and tactic or kinesthetic values that are, in a sense, the foundation of all others. This may have didactic counterparts, and perhaps the re-crystallization or re-definition of visual gestalts may call for a rather special or emphasized early role for free sculpture, especially for those caught up in issues of basic adjustment. Primitive art is especially sculptural, and retains a related health, (that Gauguin admired and taught us to see).

Symbolic stages orient the many other art phases, as theory orients experiment. The differentiation process opens out in characteristic cycles of task formation, in all of whose phases a complex modern society partakes.

The mathematical nature of ordinary life is seen also in the orienting place of hatred. Hatred masks the emergence of significant new insights. The X, once entered into, cannot be compromised, but is pushed on, ruthlessly, to self-dissolution and reformation. War is a rather incidental outer-reflection of this process. We must push blindly on one another until we reach principle. Courage is a push, not to domination, but to principle. The mathematical nature of the heuristic, in its subjective counterpart, is protected by this push to extremums. It is only the extremums that hide mathematically elaborable sources of regulation. It is the pressure of this push

keeps the bevy of over-generalized heuristics in line till their relatedness becomes more formal and systematic. It is this push to extremums that brings in the new.

The old machinery of state, of war, of poverty and pomp, terrible though its inadequacies may have been, has yet far more hidden rational aspects than people realized. Even and especially war (see report from Iron Mountain), but also the other sources of outrage, were in fact near optimum design features to minimize pressures and create needed transferences. A rule must be mathematical to be defined at all. "Good will" etc. —quickly slides off into ambiguity and hence uselessness in elaboration. The end is already one, there is no need or place for more of them, We have only to clarify the old functions in the modern research context, wherein their significance is transformed. War can then be seen as trivial compared to the quiet horror of wasted lives. "Power" is seen as also a curse and an illusion.

Harmony is a symbol of worst and best. There is the harmony of the old vision, snobbish and frantic. There is also, however, the harmony of the new, which is yet simple, and all-inclusive enough to see the uses of the old, and use it. Such harmony is mathematical, at both ends, and the hatred that leads between, marks the hard mathematical "contradictory" nature of the transition.

The relation of mathematics to ordinary life is clarified as one of question asking, (not answering). This question asking is the struggle to bring the felt within system, which has immediate counterparts in the evolution of aesthetic sensibilities, and more concretely in the self-consciously "mathematical" nature of certain "synthesis" phases of the opening of art forms. The ruthlessly mathematical nature of being (in Augustine's phrase) is ultimately reflected in the hate carrying to the point

of self-dissolution, which covers the X. Only the causal extremums persuade. The push is to the limits of felt awareness or being. That this push to limits should not be merely dissolution, but ever also rebirth, enlarged, is a perennial miracle.

2. THE UNIVERSAL ALGORITHM

The world can be viewed as a giant, or all inclusive, algorithm, generating the ordering of words. That this individuated ordering is ultimately a reflection of unqualified universals, is a very surprising thing.

First it is to be noted that we are treating the word itself as existential, although not subjective. It is a carrier of position in abstract linear ordering, other types of factors (e. g., sensations) being oriented via their effect on this ordering. The word's nature is not directly known, and not fully knowable. A word's mechanical status is made conscious only in and as the heuristics in which it partakes collide, so that certain edges of stability must be charted, or are implicit in the machinery of reconciliation.

The word is an implied unit, implied by the felt, but not itself felt. The set of negations come in groupings that define as their principle isolate these excluding channels, which orient the negations. The regulation by successive contradiction, has, at its core, the brain rhythm of successive facilitation and inhibition. But these patterns are ultimately unwound or oriented only in relation to implied words.

The word is ultimately a group-defined entity, as it is the interaction with others that constitutes the basic sources of orienting RS. Insofar as RD's (and associated implied words) are merely personal, they lead rapidly to head-on collisions

in elaboration. Lasting change has its counterpart as role change, as change in the consensus, (though not all such change is lasting).

The subjective operators are, in a sense, incomplete. They can be interpreted or read out as predictions of future orderings of our behavior, but there ever remains a gap, some of it more, some less significant. The felt represents the depth to which "we" have penetrated the world algorithm, that generates our behavioral ordering; it represents the depth to which we have reduced this ordering to repetition.

As such the felt has the status of a meta-model, predictive of the actual ordering, and generating this ordering to the extent that it is social, or realized. Physics models are part of these systematizing meta-systems, or one grouping of them. (That the linkages form a system of heuristics that, as it becomes self-consistent, becomes possessive and effective, is closely related to the sense of personality, and to belief in God.)

This character of the felt, in its relation to information processing, and reason, is a highly restricted one in many respects. Past and future enter as part of the indexing of presently effective universals, defined in terms of effect on ordering of words. This loose recognition has been formalized in a variety of ways, which can be shown to be equivalent, by A. Church, A. Markov, and A. Turing. In Church's perhaps more basic formulation, he introduces the notion of a general recursive function. A function is said to be number-theoretic when it is a function from the natural numbers onto the same set. Such functions have a finite self-completeness (unlike, for example, the operation of taking a square root). They can be set up using finite elementary tables, mechanically applied. Recursive functions are functions defined

recursively, or by induction. Shown how to get the "next" one from any given one, the whole sequence is regarded as defined.

A function is said to be general recursive, when there exists a system of (number-theoretic) equations that define it recursively. (Thus the square-root operation, can be defined by means of the familiar recursive procedure used in manual calculation of the square root. These procedures can be easily defined by means of a series of recursive equations, which are functions of the previous remainders, etc.)

Now Church's thesis says that every effectively calculatable function, (or effectively decidable predicate) is general recursive. That is to say, roughly, that all reasoning processes must be built up out of the repeated application of elementary linkages of a number-theoretic type. All thinking is just a compound of elementary word linkages of this sort, (See Kleene).

This rough intuition, more carefully formulated, becomes a very powerful empirical thesis as to the nature of understood situations, or defined situations. It can not be proven, but the rich variety of independent formulations, helps vouch for its stability and central significance.

One aspect of the definition has a certain special interest here. Induction is not involved in the definition directly (Kleene, P. 274), but rather emerges only in the application of the definition. The way in which induction enters is endless, and is not directly characterized or limited ahead of time.

We have, earlier, seen a somewhat related trick used in our treatment of the denotative grounding. We accepted the logicistic position to the extent that we

recognize that no meaning is tied down until tied down denotatively. But the methods for doing this are endless. This is an empirical result, not a straitjacket limiting mathematics. It takes endless forms, which can in no way be limited ahead of time.

Induction, similarly, defines the way in which the realm of "universals" relates to particulars. This "way" is not itself something that can be delimited. It is something that is in constant flux, as the set of universals is itself differentiated. What is orienting is not the "way, " but the fact that there are a set of universals (number-theoretic equations) which, in combination, in some given application, can be brought to bear to define the situation by recursion.

A universal (like or as a state of mind) has its own conditions of activation, where it comes into play. As these differentiate they are continually requalified; their conditions of coming into play, change. The theory of interacting game heuristics helps us to understand the dynamics of this realm of universals, which is variously compounded to reach the particular as a set of "strategies" defining game moves. This relation to the particular cannot serve as a useful point of limitation. Induction occurs here every which way.

Turing's formulation of these insights brings out other important aspects of them. In performing a computation, we can use only a finite number of distinct symbols. We also, can consider only a limited number of occurrences of symbols at a time, the number of pre-arranged instructions must be finite, and their application can only lead to finite changes of symbolic sequences at each change. Thus we can add or erase symbols, change our point of observation, and change the now effective instruction, in accordance with the previous instruction, but that is all.

Turing analyzes the acts of a computer into a minimal set of such "atomic" acts, plus instructions. The computer has an infinite tape on which it can write down or erase symbols. It has under observation one symbol at a time. It is allowed to write down a symbol, erase one, move its observation point one forward or one back, or stop, and allowed to move to a new instruction. An instruction tells which of the above operations to undergo, as a function of the symbol under observation.

This rough description is a way of trying to express the quasi-mechanical nature of a defined calculation process as it goes from step to step. When carefully formalized, it can be shown to be an equivalent of Church's insight.

Both are ways of reflecting, from the mathematical side, on the fact that the intuitive or felt grounding only has a clarified meaning to us, in and as it has a well-defined relation to the generation of a verbal ordering, well-defined by finite means.

Markov (see Curry, P. 70) develops another formulation, closely related to Turing's, in which one has a set of commands, transforming certain factors in expressions into other factors. Church orients the universal linkages more abstractly, (elsewhere relating them to abstraction processes). Turing emphasizes the application process in mechanical detail. These various ways of clarifying the relation of the realm of universals, to the concrete linear ordering it determines or shapes, has useful counterparts in various philosophic techniques describing God's relation to the world. The ultimate equivalence of these, at first, seemingly quite different approaches (emphasizing abstraction, or the realm of universals viewed in itself, or the machinery of application, etc.) helps to dramatize the centrality of the isolate here variously subsumed.

Actual felt categories are more closely related to the dynamics of resolving conflicts in such clarified procedures, but the ideal of such procedures orients the situation in indirect but effective ways all the same.

3. THE META-SYSTEM

The implied universal algorithm is unknown to us. What we "know" is a sub-aspect of this which is a function of the class of repeating paths that has emerged, and our related ability to predict ahead far enough to isolate them, (since to know them is to enact them).

The subjective has thus the status of a meta-model predictive of actual ordering, but often ambiguous or at variance with it. We have here the unusual situation of a meta-model which is smaller than the system it models. The meta-system is (in part in consequence) always full of latent X, which elaboration brings to the fore. Such a system is none the less useful, because our concern in "predicting" the universal algorithm is a limited one. We are only concerned with breaking active inconsistencies in the scheme, not with the wide areas of behavior which chance, etc., are left free to shape.

The X in effect creates competitive abstractions, it creates that conscious wish-like gap between expectation and occurrence. The over-stated branches, as they become reconciled to one another, become also socially effective in shaping behavior or pushing deduction into wider areas. With reconciliation, the meta-model-like view, becomes, in a limited sense, part of the basic generative algorithm. One has what could bear analogy with the sort of correspondence between higher and lower

levels that is used to simplify the logicistic position, an only rough correspondence, which here helps illumine the language of union with God.

It is the latent X in the readout procedures that creates the abstraction, or is one aspect of the over-statement with which we all start. This X is first denied — in the harmony which represents the worst. Then it is enacted, as choice of sides — placing the X outside ourselves on others. Later it is internalized as self-collision — placing it in ourselves. Finally, it is resolved, or isolated as former syntactic error, with a return to a (relative) harmony. In this harmony we pass out of the past into the future (Thomas Wolfe) and the wish-like meta-models become effective generators of social order, or pass, as it were, to a more active (non-meta) status, in the read out.

The X focus the opening up of the universal algorithm, exposing the historical roots of the heuristic-like elements that compete. It forces the need to objectify the heuristics in collision, to break them up into person independent components.

It is this alternation that brings the system-like character of the universals that generate the word ordering, into explicit play. The reconciliation of competitive heuristics goes via the clarification of its person independent components. As a heuristic, and part of system, it has the status of a theorem that is deducible, and leads to further deduction. The clarification and stabilization of its position involves the exposition of tree-like components of descent from a minimal set of independent unqualified forms, or person independent forms. The "shell" of consistently or mutually supporting links is very sensitive to one that does not "fit. " It forces a disintegration of ordering factors—and consideration of ranges of ordering wide enough to catch the descending invariants ever hidden in colliding memories, —

when these are strong enough to cause self-dissolution at all (by their excess). The proper symbols (number of needed independent words) and their allowed orderings, etc., are all properties of the given situation, which come apart at different places, depending on the nature of the X.

Everything that effects a given X is brought to bear on it. The basic resolution is an identification of the RS or SRS component in question, within a system, and hence, as part of a substitution game (Turing). The SRS criteria and RS are then further clarified as derived from the collision of competitive consistency criteria of a common system. Solution is a further qualification. The perceptions of identity that underlie the competitive heuristics are formulated within common fields so that the problem can be given to "scientists. "

Reconciliation is made possible by mathematicalization not threatened by it. In so doing the X or felt moves elsewhere, and there is a return simplification which is most complex (Brancusi) and not deliberate, but paradoxical. Tying down feeling denotatively lets it migrate elsewhere. It permits of the simplification called warmth.

Because the X is felt as X, also, there is a sense in which the "full" readout is never in X, the X being ever viewed from without. It exists in a sense only in the meta-system. But from the point of the basic system, this "X" is not an X, but only a pointer to the enlarged system it helps uncover.

The problem of the self-producing machine is bypassed in this game-theory type method of analysis, by what in a sense is a kind of trick. Starting in effect with a set of potentially actionable universals defined with respect to the linear ordering, as

parts of heuristics of the common goal function, we have the self-production built into every component from the start. The task is to take apart these components bit by bit, till they are shown as compoundable of person independent factors (i.e., denotatively rooted). We make the reaching of the particular the "problem" (instead of reaching the institutional) somewhat as the retroactive inhibition theory of memory makes forgetting the problem, not remembering.

Von Neuman shows that a self-reproducing machine could be designed (Kemeny). Where one can be designed, an infinite set can be. A paper by McCulloch and Pitts showed the potential adequacy of the synapse to represent all logical structures, and so shifted the question from "the whether or not" to "which" synaptic structures were the ones. So, too, here; — the problem becomes "which self-reproducing machines are the human ones? " As such machines are of a very special type, it is not surprising that a very special mode of analysis (game theory) is adopted to take advantage of these most peculiar symmetries of the machine, to simplify our thinking.

Hillary Putnam points out how Turing machines (or any calculating machine) forms an ideal setting within which to reformulate or concretize the question about the relation of logical structure (felt structure—etc.) and mechanical structure. When we do this the old simplistic, "merely" philosophical arguments about the nature of their interrelation, are quickly seen to be inadequate.

But the old questions are not meaningless on that account. When we set up a logical structure in electron tubes, the logical structure is there as well as the tubes. One could, if he likes, say the tubes were not there, that they were merely fictions, that

what was "really" there were "only" electrons and protons. What is "really" there is a question of the problem in hand. Many things are there. The problem of relating subjective categories to hardware categories is very much like that of relating hardware categories to logical categories they embody. It is a complex matter, and can't be settled by any philosophic truism. But, on the other hand, it is doable. There is nothing self-contradictory or absurd in the aim. And when it is done, a rich variety of symmetry properties are revealed that expose latent depths and near-misses in the earlier philosophic over-simplification, that ridicule is apt to blind us to. To "identify" the felt with certain mechanical forms is no more (or less) a confusion, than to identify a piece of steel as macroscopically defined, with large number of smaller atomic units, arranged in certain ways.

The theory of games forms a framework in which to clarify the relation among these various categories. A "heuristic" of the common goal function is a mixture of logical and nature-calculator factors that groups what is in external terms a rather complex array of interacting factors under a common head. It exposes the relation of meta-model to model, of logical to mechanical categories.

4. THE PLACE OF LOGIC

a. The Formalized Language

The concept of a mathematical system, is a kind of mechanical substitution puzzle. It is made of proper symbols (with effective rules for their recognition) rules for well-formed sentences or for allowed strings of proper symbols, and rules of substitution, rules for getting from one to another string.

The concept of a mathematical system gives no special place to those sub-systems called the propositional or predicate calculus. Yet these sub-systems are almost universally present in mathematical systems of any size. Why is this? What is the origin of the special place accorded these sub-systems?

Any mathematical system can be easily so enlarged as to contain these sub-systems, so that these requirements do not represent limitations. Rather their importance reflects upon the human element in all understanding. The enlargement associated with the propositional calculus reflects upon the fact that all distinctions, as understood, are, or can be tied down in person independent, or "denotable" distinctions. It reflects upon the multiple independent links with which any fact is tied down, once caught at all.

The enlargement more narrowly associated with the role of the predicate calculus is related to the second linearization (as the propositional calculus is more closely related to the first linearization). To systematize the second linearization, we view the denoted categories of the first from the standpoint of creation, something possible at all, only because the world is the embodiment of unqualified universals, via which the multiplicity can be indirectly treated, (in a substitution model).

The process of making consistent, or reconciling, via the tying down in (generating) truth functions, is a very delicate probe for latent X. The importance of the X in orienting the thinking process, thus reflects itself indirectly in the importance of truth functions that further orient the X, (another reason for the prominence of logic in mathematical systems).

The basic orienter of meaning is this naming process. As motor patterns differentiate, they isolate person independent components that are communicable or nameable. This is an experimental isolate, a kind of halfway house, or middle region. The named categories, when in conflict, must be further demythologized, exposing their relation to the "second linearization, " that is, the effect they have on our behaviors, their place as part of a set of heuristics. But for the most part we do not need to do this, and live comfortably enough in the middle region.

In what is most vital to us, in what defines our character, we can not yet reach to denotative roots. But even here, the ideal of so doing, orients.

Logic derives its central importance from this experimental emergence of a person independent root, the named patterns, within all clarified issues. "Logic is the analysis of sentences (or propositions or proofs) with attention to their form, in abstraction from their matter. " It is a surprising thing that there is such a common form to be found at all, beneath the caprice of feeling, and personal difference. That it can be found (after suffering) is closely related to what the scholastics called the clarity and simplicity of God.

We will quote a few basic definitions from Church's book on logic, with an eye to pointing out certain features that have a broader relevance for exposing the relation of subjective to syntactic categories. Such few comments as are here (and elsewhere) made on logic, are clearly not self-contained. They imply some reading in modern logic, but in addition are crudely amateurish at best. If they motivate professionals to follow up on latent, broader meanings of logic, in the context of the NS model, they will have served their purpose.

"The name is always the name of something. " This, of course, is a restriction on the meaning of name, and in part semantic. But that, as a restriction, it can be imposed, without throwing out, or violating any ultimate meaning, says a great deal about the world.

"That which the name names is called the denotation. " "The sense is what one grasps when one understands a name." "The sense is a concept of the denotation." "The relation between a proper name, and what it denotes, is called the name relation."

Ultimately this understanding of a name and name relation, this trinity of name, sense, and denotation, are clarified by example, but can not be themselves "linearized, " (or defined operationally) any more than the concept "matter" can be. In any given or limited set of situations, operational definitions, or models of the naming process can be devised. But it is only too easy to get around these, to give forms of naming that are overwhelmingly adequate in practice, that do not come under these operational definitions.

Every opened layer of the structure of matter carries with it a new self-concept, new more powerful consistency criteria, and associated ways of making objective or person independent, whole classes of feeling that were not previously communicable. What is surprising about the naming process (as Weyl in effect points out) is how overwhelmingly adequate it is in practice. In theory it may seem a highly controversial and speculative matter, but in practice, when two scientific theories collide, it is never because there is a conflict over what is meant by naming, but only because that named is in disagreement.

The complex patterns of naming have now to be studied, and abstracted from, as the basis of orientation of other types of forms, to clarify their inter-relation. All clarified or understood meaning can be tied down in denotation. (It can also be tied down in other ways. The assertion of one all-inclusive potential method, does not deny the presence of others, as, for example, "effect on ordering of emissions, " and "comparison of lengths," etc.) Let us consider some elementary steps in building up complex names.

"A constant is synonymous with a proper name having a denotation. " "A variable is a symbol whose usage is like that of a proper name, except that a single denotation is replaced by the possibility of various values of the variable. " "A variable has associated with it a corresponding non-empty range of possible values called the range of the variable. " "A form is a complex name, one or more of whose constituents is replaced by a variable. "

So we begin to build up webs of inter-related names. These inter-relations run themselves under endless headings. (Y's uncle. Z's color, etc.) It might seem that we are just heading off into a swamp.

What is orienting here is the abstract inter-connectedness. Ultimately, via the model of the nervous system, we are going to be exposing the relation of logical forms, i.e., this abstract inter-connectedness which is the syntactic end, in its dynamic aspect, to logical content, the felt categories (for the present ambiguously pinned down in categories of sense and denotation).

This abstract inter-connectedness is partly isolated off via the very general notion of a function.

"By a (one-valued singular) function, we understand an operation, which when applied to something as argument yields something as the value of the function for that argument. " "Two functions are identical (in extension) if they have the same range, and have for each argument in the range, the same value. " "The function concept is like the sense of a name. Two functions may be identical in extension but based on different function concepts. "

Any form, as previously defined, has an associated function, derived from it by abstraction, (though the reverse is not generally true). By pushing to the limit slowly this process of abstraction we distill out, as it were, a very small, merely "human" element, an element closely related to information processing and the will. What is surprising is that as we expose the relation between subjective and syntactic categories, this rather special aspect, turns out to contain latent in itself, all aspects. (The course is an attempt to start the tracing of this inter-connectedness in its major outlines.) Or, more correctly, it is discovered (experimentally) that the inter-connectedness isolated by abstraction, is something extended to the felt by our own activity. The attempt at self-conscious isolation of any felt component, is its destruction, as its stability is a dynamic matter depending on a relation to wide classes of latent behavior. "Nature" as felt is an emanation of the will.

Now the concept of a formalized language is a powerful source of orientation of the theory of meaning. In a well-formed, or formalized language:

1. Each name has only one denotation.
2. When a constant name is replaced by another having the same sense, the sense of the entire name is not changed.
3. When a constant name is replaced by another having the same denotation, the denotation of the entire name is not changed.
4. The denotation of the name is a function of the sense.

Most verbal usage is not inside a formalized language, and the attempt to put it inside directly would often be a useless if not impossible task. Nevertheless, the formalized language provides a powerful ideal orientation. In any specific conflict, for any limited issue, the requirements of the formalized language can be met, as far as is in practice needed. The sliding in of the maze of qualifying variables, needed to bring things inside the formalized language, is not destructive of meaning as it bears on any given application. It is a requirement that can be met in principle, and carried out in practice insofar as really needed and useful, (a very limited need).

The very process of probing the other person, getting him to tie down what he means in denotation, is a deeply emotional matter, and highly enraging. When situations are not adequately probed, forms of blind mutual rage are needed to keep the information processing going at all. We all love our hates, — we could, for the most part, not function without them. Like war (which is their outer reflection) they are a very small price to pay to help loosen the local stagnation, the local suffocation, which the elaboration of the word brings on. Death by war is but a fly-bite at the side of the quieter death by inner rot and suffocation that takes the rich potential lives of all about us.

The test, is whether or not we can establish, using words, the sort of symbolic completeness (after suffering) that makes the denotative root adequate (to our present task). Whether such a transcendence of loathing is practical, whether it results in more than a boring sleep, is yet to be proven. Certainly our hatreds have been wonderfully fruitful devices, defining character.

Belief in the formalized language, or its orienting place, is closely tied to a belief in the (relative) transcendability of hate, and the ability to live in the "suffocating nightmare harmony" that results, — indeed discovering it not so lazy, or so trite as we had supposed, but (by grace) dynamically opening. It is only slowly that we can give up our loathings, — our rejection of others, which simplified the IP. But the best evidence is that it can be done. Factuality is not "detached." In "detachment" one touches no felt category at all. It is only in drawing back together all the hated and rejected elements of the community, that one digs up a set of denotative roots, powerful enough to allow of decision, without loathing.

This digression on the emotional significance of the concept of a formalized language would lead far afield. It is touched on here because it is important to keep in mind the emotional counterparts latent in certain assumptions, or we will make them in one phrase to ignore them in the next.

Underlying the concept of the formalized language is a very general theory of incomplete symbols. Symbols, not themselves names, can always be treated as incomplete symbols, which, when compounded with other symbols, define names. The theory of incomplete symbols inside a formalized language, is complemented and enlarged to include all language in this way.

Words not a part of the formalized language, are, in effect, all treated as incomplete symbols, which, when further qualified (combined with other symbols) enter the formalized language with no loss of essential meaning, essential meaning relative to the (i.e., any) given issue, that is.

"Therapy, " in its broadest sense, is just a technique of completing the form. But, as thus viewed, therapy is all of life, not part of it — all science being an attempt to remove the psychic pains, or neurosis, that latent X induces in different places.

b. Truth Functions

We have come this far with no special mention of truth functions, or propositions. One reason for this is that (with Frege) we are treating truth and falsity as names of two abstract objects, much as we treat any other name. This represents an important departure from classical logic, a departure of deep significance, which calls for explicit discussion.

All categories are, (as felt), transcendental. First matter is transcendental, the self-concept shares in the transcendental nature of matter, denotation shares in that of the self-concept, and via these, or as aspects of these, all other felt isolates become transcendental, too. If we cannot build a general model of what we mean by truth or falsity, neither can we of anything else. In any given situation we can, and in doing so we are treating the condition truth or falsity on the same footing as a tree or road. We are doing away with its special, mysterious, "meta-physical" (?) status and bringing it down on a level where it can enter into relations with other categories on an equal plebeian footing, as a name among names.

To fail to treat the meaning of declarative sentences as part of the theory of names, just introduces a duplicate of all the previously developed machinery, for no good or apparent purpose. The theory of the meaning of sentences would be isolated off from that of the meaning of names, and we would be faced with the formidable task of justifying this isolation in the modern context.

Once sentences are regarded as types of names, the earlier mentioned properties of the formalized language, quickly force us to Frege's conclusion. Principle "2" tells us that "Scott is the author of Waverly" must mean (in the formalized language) the same as "Scott is Scott. " Or "The number, such that Scott wrote that many novels, is 29" and "The number of counties in Utah is 29, " must also "mean" the same. What is common about such different sentences is only that they are true. "Elaboration of such examples leads quickly to the conclusion as at least plausible, that all true sentences have the same denotation. "

"Therefore, we postulate (with Frege) two abstract objects called truth values, one of them being truth, and one of them being falsehood, " (similar arguments as the above holding for all false sentences). "We declare that all true sentences denote the truth value truth, and all false sentences denote the truth value falsehood. "

"Any concept of a truth value we shall call a proposition. " "A proposition is true, if it determines or has the truth value truth, false if it has the truth value false-hood. " "A variable whose range is the two truth values, is called a propositional variable. "

Now we have done a lot of dirt to piles of philosophic insights into the nature of truth and falsity in thus re-defining things. A great many very powerful insights into the dynamics of the operation of the brain, were, traditionally, introduced as discussions of the meaning of truth and falsity. What we see taking place here is a pattern that repeats itself endlessly as models differentiate. Old intuitive notions of great substantive power are rejected, and replaced by trite notions defined, instead, directly in terms of calculational procedures. Preservation of rules of calculation, simplification of information processing schemes, is regarded as a more "basic" guide than the intuitive insight, — (e. g., the advantage, above seen, of treating sentences as we do names). In so doing, however, we lay the ground work, for re-creating the old intuitive insights, as vastly more powerful technical insights. The old significance of truth, its relation to the will, its relation to consistency, are all expressed now (via patterns of names) as basic laws of operation of the nervous system. Thus what is in a sense a step "back, " becomes a big step forward. One must not suppose that these re-definitions of truth are deeper or more correct than the old ones. Their value derives from their trivial superficiality which helps provide a foundation in which the old depths can be explicitly laid forth, (e. g., the relation of truth to assertion, etc.).

That naming should be treated as more primitive than assertion, (assertion being a special type of naming) has a kind of natural relation to Kierkegaard's observation that thought or self was vaster than the real world, the real world being defined as a sub-aspect of self, via the category repetition.

Another dimension of truth is its two-valuedness. This is related to the objectivity of the world, and has very deep roots. Three (or infinite) valued logics may

be interesting mathematically, and even serve some purpose, but they offer no substitute for the orienting significance of two-valued logic. We coordinate the "concepts" of higher valued logic with two-valued factors when they are well defined, and when states are not well defined the complex relation of the unknown to the known never slides within the simplistic three (etc.) valued molds, but involves contradiction, complex uprooting, and reformation.

Now this naming machinery, with truth as one of the objects named, is a framework into which all the forms of language in common parlance can be made to slip (under the threat of a useless and confusing duplication of language if and where we opt not to let slip).

Basic to this interpretation of common parlance inside logic is the treatment of classes as singularly propositional functions. The class "Uncles" is the set of objects for which "X is an uncle" denotes the truth-value truth. Associated with any class is a related propositional function which defines it, and via which it can be treated. The name pattern isolates a set of correlations in a person independent form. Their "assertion, " finds them part of the system of consistent linkages called truth, much as the recognition that Scott is the author of Waverly, finds Scott in the system of linkages involving authorship of Waverly.

Naming isolates the correlation. Once isolated, there then ensues a long process of digestion, during which the "qualities" used in isolating names, must be themselves demythologized. This is a concern of science. Logic stops at the first caught stage, (where system is defined enough to define problem).

Relations, (like classes) may be brought within the logical scheme by associating them with binary propositional functions. A variety of other problems related to the translation of ordinary descriptive language into a formalized language are discussed by Russell (Introduction to Mathematical Philosophy, Ch. XVI). Of course, there is a great deal of arbitrariness in the details of just how one sets up such a language. But such arbitrariness is in the main a question of convenience, and does not affect the basic insight as regards the orienting place and significance of the naming function.

Definition in intension versus definition in extension reflects the rich variety of ways in which a same syntax may be tied down. From a formalistic point of view, there is a syntactic core of rules and linkages which is defined independent of any such distinctions. A set of things, or the operation of enlarging a set of things, may, either one, be regarded as "existential, " leading to different sounding interpretations within which different syntactic rules, or the same syntactic rules, can be rationalized at will. The logicistic position is not, as such, committed to any "type" of linkage. (It is so easy to Platonize Aristotle.)

c. On the Propositional Calculus

Let us glance momentarily at a bit of the machinery of building up of logical systems. Proper symbols are the names and variables. Improper symbols, (i.e., other symbols) have no meaning in isolation (as do proper symbols) but combine with other symbols in such a way as to determine or have a meaning. The theory of improper symbols we are extending beyond the formalized language, via, what could be called, a theory of incomplete symbols. Symbolic usage, outside the formalized language, we

are saying, can always be brought into the formalized language by a process of introducing qualifying variables, etc., splitting up and clarifying latent meanings, at least insofar as this usage is understood, at all. In a transcendental world, oriented with respect to the ingress of new truth or change, there are, of course, always vital aspects of the present situation not yet ripe for such a reduction, and it is important to realize this. But the potential reducibility remains a deeply orienting guide.

Connectives are combinations of improper symbols which can be combined together with one or more constants to produce new constants. (Operators are combinations of improper symbols which may be used together with one or more variables (as well as constants and forms) to produce a new constant or form.)

The basic logical connectives (such as "and" and "or" and "if, " etc.) are defined, (indeed often radically redefined) in the context of modern logic, with an eye to simplifying the operational procedures. The first, and major simplification that is imposed on these old words, is that their meaning is regarded as entirely subsumed in what are called truth tables. (See Ambrose for a good elementary presentation.)

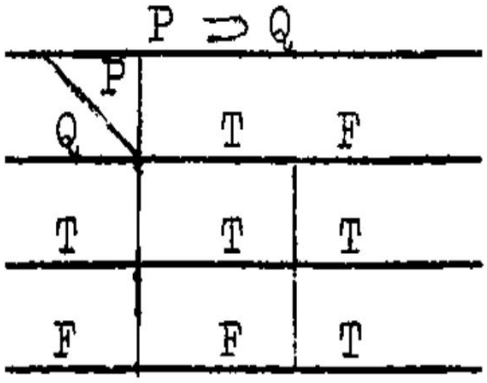
Thus "P and Q" (often written P. Q) is a proposition that denotes the truth value truth only if both P and Q are true, else it is false. Similarly "P or Q" (often written P v Q) is regarded as true if either P or Q (or both) are true. In common speech "or" is often defined to mean "not both." This usage is not adopted. "P implies Q" (often written P ⊃ Q) is true if P is false, true if P is true and Q true, false if P is true and Q false. That one should speak of an implication as true when the first part or assumption is (always) false, again serves to emphasize the purely formal character of these definitions. Implication, classically, had infused into its meaning aspects of the

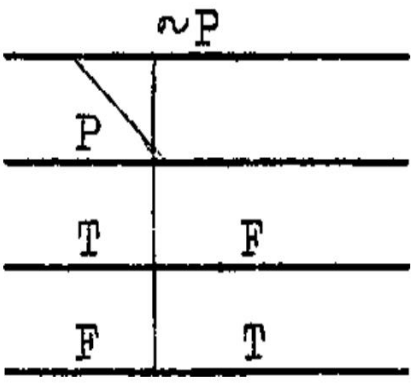
laws of operation of the nervous system. It was, of course, related to the conditioned reflex principle, as a subjective reflection of the elementary "felt'' correlation that obtained between facts both true at once, their tendency to induce each other in feeling. In addition, the classical discussion often reflects on the primary place of such implication, and the secondary role of negation. Pavlov makes the same basic observation, noting that the positive faciliatory links are more stable, and far less changeable than the regulatory inhibitions.

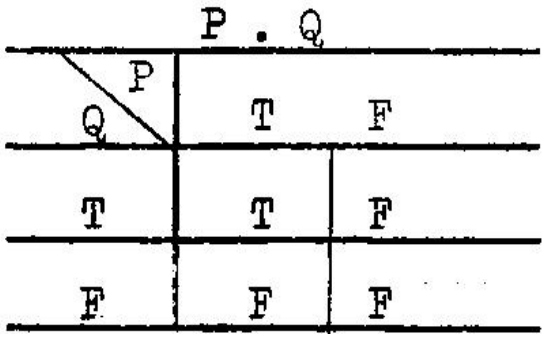
The relation between the properties of inhibition and facilitation in the organization of the brain, often sensitively reflected on in classical logic in process of discussing the meaning of truth, have, of course, no place here.

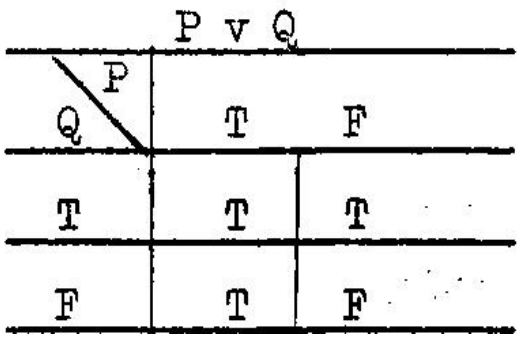
Another connective of importance is negation, or "not." "Not P" ($\sim$ P) is true when P is false and false when P is true.

These points are subsumed in the following tables:

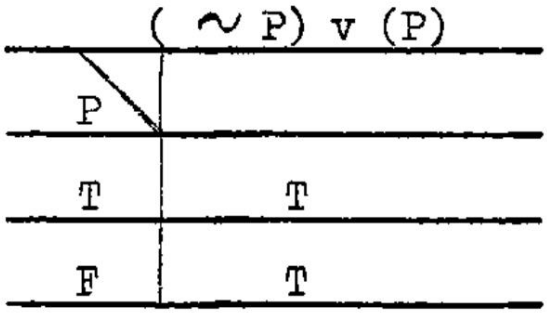








Other logical connectives can be easily written in terms of these, (or fewer than these).



Some propositions can be seen to be always true, from their very form, without regard to the truth or falsity of their constituents. Thus, for example, ((not P) or (P)) is always true.

A tautologous function is a function which denotes the truth value truth for all values of its arguments. It is possible to view all mathematics as concerned with systematizing the class of tautologous functions. (To be tautologous is not necessarily to be obvious.) Such a point of view is rather special and a bit artificial. If there is no interpretation given, then who is to say? With sufficient ingenuity, we can so interpret

any syntactic system as to view it as a set of tautologies. This may be and is an interesting point. Though there is nothing that compels us, it provides a kind of orientation. Doing so, helps isolate the "mathematical" part from the "substantive" part of a system.

The propositional calculus is in a sense the most elementary and universal system that underlies, or forms part of all other major mathematical systems. It can be viewed as a system for the derivation of all the tautologous functions that can be built up out of these elementary connectives. A brief glance at its form will give us an example of a mathematical system, or substitution game, to keep in mind as an illustration of earlier, more general points. (See Ambrose)

Mathematical systems can be regarded as made up of three parts, as we have said; — rules for proper symbols, rules for well-formed sentences, and rules of substitution.

Well-formed sentences or sequences can be formed as follows in the propositional calculus:

1. "P", "Q", "R", (etc.), are truth functions.
2. If A is a truth function, then ~A is a truth function.
3. If A and B are truth function expressions, A v B is a truth function expression.
4. Nothing else is unless it is replaceable by definition by a truth function. (For example, P ⊃ Q is, by definition, the same as (~P) v (Q) and may be substituted for it.)

The rules of substitution allow us to start with, or add on any of a set of postulates. The following might be taken as such a starting set (though a smaller one would also do).

(P v P) ⊃ P ---------- Principle of Tautology

Q ⊃ (P v Q) ---------- -- Principle of Addition

(P v Q) ⊃ (Q v P) ----------Principle of Permutation

(Q ⊃ R) ⊃ ((P v Q) ⊃ (P v R)) -------- Principle of Summation

(P v (Q v R)) ⊃ (Q v (P v R))--------Principle of Association

The following basic rules of substitution then allow us to derive all tautologous propositions that can be formed from these connectives.

1. We may substitute for any variable (P, Q, --) any function definable within the system. If what we started with was a postulate or established theorem, the result is, too.
2. Any expression can be replaced by one which is definitionally identical with it.

(The following are sometimes called rules of assertion)

3. Given P, and P ⊃ Q, we may infer Q. (The principle of inference.)
4. If P and Q are postulates or proven theorems, then their conjunction (P . Q) is also. (The principle of Adjunction.)

Although the propositional calculus is a very restricted system, not allowing room for us to speak of totalities, etc., nevertheless it plays a very central role. In application we are only concerned with parts and finite matters. The infinite is something of a lie, introduced to simplify the information processing, only to be taken away again. The whole is never explicitly relevant as such. Other systems as enlargements of this system, become special cases of it in routine application.

d. The Quantifiers, (etc.)

Feeling is a universal, yet that in terms of which it is represented is an implied particular. The infinite is the device used to bridge this gap. To represent universals as felt (and hence without their qualifications) one is caught up in the infinite at every turn, only to remove it again as the felt abstractions are compounded to define a definite ordering or practice.

When totalities are not limited, then the so-called quantifiers, "there exists" (∃) and "all" (∀), or "the Y such that" (iY) play essential roles, that cannot be replaced or defined in other earlier (finite) terms.

They carry with them a whole new set of rules of substitution, which in finite cases could be derived from old rules, by defining ∃ , ∀ etc., in terms of the specific cases, but cannot now. Part of these rules are the Aristotelian syllogisms, somewhat streamlined to simplify the rules a bit, but basically the same. There are four elementary types of situations;

1) "all X is Y".

2)"Some X is Y".

3) "no X is Y".

4) "some X is not Y".

(1 implies 2 and 3 implies 4.

1 contradicts 3 and 4.

3 contradicts 1 and 2, etc.)

But in addition to such enlargements there is the need to distinguish between free and bound usages of variables. When ∃(X), or ∀(X), occur, the X's in what follows are no longer "free. " One cannot replace them by specific values from their range without making nonsense of many statements, and leading rapidly to contradiction. Rules of substitution become correspondingly complicated as one must worry about bound and free occurrences. Such things make the so-called predicate calculus much more complicated than the propositional calculus. One needs to worry about the order of complexity of a form, whether it involves a simple function, or a function of a function, etc.

Broadly speaking, this more flexible machinery dealing in terms of totalities, via generating or defining qualities allows us to get at our feelings much more intimately in complex situations. Where the old heuristics are in X, qualifying variables can enter by induction, out of the past history supporting the conflicting paths, in endless ways. The place of the verbal correlation, as of the qualification in the information processing that can stop the X, is apt to be at any stage. We cannot build a limiting model of induction.

In a sense we live in the "infinite" and have the illusion of not doing so because of the jumping about of the X, which is treated second hand. To inter-relate the forms of the X, before reconciliation, we project them out as parts of the so-called real world. Later (in Tillich's phrase) we find that the absolutely concrete is the absolutely

universal, and that the rules hidden in the statistics thus spread out, have a simple subjective correspondence as an enlargement of the realm of generating universals.

The adequacy of this passage back and forth, using finite models, is brought out by theorems of Lowenheim and Skolem. (See Kleene, P. 426). Roughly speaking, any system (that can be formalized in the first order predicate calculus—and most of interest can)—that has a model at all, has an enumerable model, and if it holds in all these enumerable models, it holds in the whole system. "Higher order" constructions or notions all are built up or abstracted from, finite situations, and this has its indirect implications on the systems forms.

It is easy to build machines to list all provable theorems, but it turns out to be impossible to give a (terminating) test to tell whether or not a theorem is provable, for if you could, you are led to certain contradictions.

All the improvable theorems according to Turing are closely related, a matter of showing that if you could do certain other things, then you could make a test to tell whether or not a theorem was provable, etc. The philosophic importance of these matters is often exaggerated. As Turing points out, we can build a mechanical "substitution game" model of these theorems, and in them the unprovable theorem becomes a statement about how one can't reach certain configurations by the allowed (rules for) mechanical changes. True statements about systems can be developed that cannot be proven within the system, —indeed in any system that is "big enough. " Mathematics is, as a whole, an opening system for much more powerful and primitive reasons than these. No system contains all systems, but it is good to note, too, that no "big enough" system can derive all true statements about itself.

If the main body of conventional mathematics cannot be proven consistent by finitistic means, (as Goedel had hoped) it can be proven consistent using freer methods. In the context of the infinite, words from ordinary practice take on new meanings. All application is "finite". There is nothing uncertain in the usual sense of the word about our mathematical tools.

Whether to accept the axiom of choice, what kinds of proofs are to be allowed, etc., are, from a formalistic point of view, only questions as to which rules best simplify the information processing. In view of the existential status of the word, and in the context of a causal self-model (or world outlook) that deals directly in verbal correlations in (what are ultimately) causal terms, the formalistic point of view takes on a self-complete and adequate grounding status. To will a mathematical connection is to give it existence, as qualifying verbal compulsion in us.

Implied felt patterns (or gestalts) open in jumps, since changes occur at the level of the generating universals. It is not a gradual matter. The bridging correlations are all hidden up in the universal. Lesser ones are not invariant enough to take hold. Hence, the power of the mathematical rule of generalizing the problem to clean out what is essential, or the parallel and deeper guide of maximizing the inner conflict.

The process of completing a form involves tying it down in denotation, or asserted patterns of denotation. When correlations (C's) are in conflict, we must open out the heuristics that underlie and push them to a deeper reconciliation (or better self-model) to resolve, i.e., reach denotation. But the X that drives this opening must first be well rooted in conflicting denotations or conflicting predictions of the old games if it is to have the psychic push needed to reach new forms of leap from

syntactic to subjective categories that resolves the X and lets us drop the over-responsiveness of the old games.

Correlations come, roughly, in three types. Denotation catches person independent substantive correlations. Mathematics catches the formal correlations implicit in our own assumed compulsions. In between in a sense are the enacted personal correlations, which we attempt to compound out of the person independent ones, using the elaborate mathematical nets in which to try to find resolving methods of identification of the outer parts.

The X's abstract off, reducing our world to just so much definition of the two branches as is needed to maintain the X, and winnow for a resolution. As the sub-X's resolve— more complex forms consolidate out, and slowly one fills back in to the present. The branches, as in X, appear detached, representatives of what they are not. As they resolve with each other they become effective, and change level, so to speak. As more qualifying variables are read in, the felt is treated closer and closer to what it "is, " (instead of as a symbol for something else). To treat it fully as what it is, is to treat ourselves as related to God by obedience.

The felt becomes itself theoretic in resolution. The alternation that all suppression entails (for choice of sides always comes in X'ed sets that cycle) gradually yields new love. To love one's self is the hardest of tasks, and it yields new forms of the state, in consummation. The old forms of rage fall off into paralysis, or indifference, they cease to be activated, as the old forms of late love lose their selectivity, and open to reunite us all. The new then emerges romantically within the forms of the new science itself, as they become possessive and self-complete.

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